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Satellite Tracking Control under J2 Perturbations

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Abstract— Inter-satellite tracking problem involving two satellites in low earth orbits is addressed here.Two satellites involved are a chaser satellite and a target satellite. The modelling of relative motion among the two satellites incorporating J2 disturbances is discussed here. The objective is to design a suitable controller to keep the relative distance among the satellites constant. A LQR controller is designed and simulations are done in MATLAB.

Index Terms—HCW equations, Hill's frame, J2 effect, satellite orbital elements.

1 INTRODUCTION

Satellite formation flying is the concept that group of Satellites can work together to accomplish one larger objective. Multiple-satellites has many advantages over single satellites and has wide range of applications in various fields.Intersatellite tracking problem involving two satellites in low earth orbits is addressed here. Two satellites involved are a chaser satellite and a target satellite.

Clohessy-Wiltshire equations are used to describe the linearized equations of motion of two satellites in proximity orbits.CW equations do not incorporate disturbances. The J2 perturbation due to earth oblateness effect is the most predominant disturbance. The earth's equatorial radius is 21km larger than the polar radius[1]. This flattening at the poles is called oblateness[1]. The dimensionless parameter which quantifies the major effects of oblateness on orbits is J2[1].The J2 effect causes a twisting force on orbit of satellite that will change various orbital elements over time. Thus modified CW equations including J2 effect is modelled.It is necessary to keep relative motion between the two satellites constant under these disturbances. An LQR control is applied to the system to keep the relative motion under J2 perturbation constant.

2 CLOHESSY WILTSHIRE EQUATIONS

Consider a chaser-target satellite close formation flight system in close proximity orbits around Earth .The linearized equations of motion of a chaser satellite that is in close proximity orbit to target satellite can be described by Clohessy-Wiltshire equations.(1)-(2).Where 'n' is the orbital

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rate ($n=\sqrt{\mu/a^3}$), a is the semi major axis of target orbit and the variables x, y, z;$\dot{x},\dot{y},\dot{z}$ and $\ddot{x},\ddot{y},\ddot{z}$ refer to the relative displacements, velocities and accelerations in the radial (x-axis), along track (y-axis) and out of orbit plane (z-axis) directions in Hill's frame.

$$\ddot{x}-2n\dot{y}-3n^2x=0 \quad (1)$$

$$\ddot{y}+2n\dot{x}=0 \quad (2)$$

$$\ddot{z}+n^2z=0 \quad (3)$$

It is a reference frame with (x, y, z) co-ordinates whose origin is fixed at the center of mass of the target satellite and rotates with an angular velocity ω with respect to Earth-centered inertial reference frame. The frame has its x-axis along the radial direction, the y-axis along the flight direction of the target orbit(in track) and the z-axis is out of the orbit plane(out track) and completes a right handed reference frame. The HCW equations in state-space form are given in (4) and their solutions in (5-6).

$$\begin{bmatrix}\dot{x}\\ y\\ z\\ \dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix}=\begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\3n^2&0&0&0&2n&0\\0&0&0&-2n&0&0\\0&0&-n^2&0&0&0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ \dot{x}\\ y\\ z\end{bmatrix} \quad (4)$$

$$x=(4-3\cos(nt))x_0+\frac{\sin(nt)}{n}\dot{x_0}+\frac{2(1-\cos(nt))}{n}\dot{y_0} \quad (5)$$

$$y=6(\sin(nt)-nt)x_0+y_0-\frac{2(1-\cos(nt))}{n}\dot{x_0}+\frac{(4\sin(nt)-3nt)}{n}\dot{y_0} \quad (6)$$

$$z=z_0\cos(nt)+\frac{\dot{z_0}}{n}\sin(nt) \quad (7)$$

The equations (5) and (6) are coupled and define motion of the chaser satellite in XY plane of the reference orbit[1]. The

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equation (6) has a term $3(2nx_0+\dot{y}_0)$ which increases linearly with time. [1]. Unless $2nx_0+\dot{y}_0=0$, the chaser will drift away from the target and the relative distance r between them will increase without bound. [1] That is C-W equation should maintain a condition $\dot{y}_0 = -2nx_0$.But perturbations may change this condition and thus affect the relative motion among the two satellites. The orientation of the relative motion under nominal conditions in different planes can be plot using the equations (5)-(7) and is shown in figure(1)-(3).An initial relative displacement, $r_0 = [x_0, y_0, z_0] = [10km, 10km, 0]$ and initial relative velocity,$v_0 = [v_{x0}, v_{y0}, v_{z0}] = [20m/s, -20m/s, 0]$ is assumed for fig.1 which satisfy the initial condition for C-W equations. The relative motion among the satellites under perturbations is shown in the fig.(4)

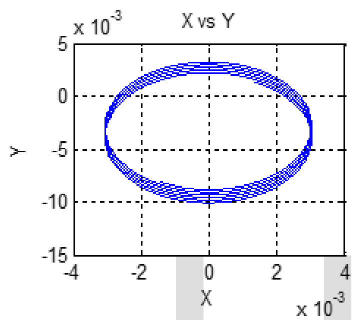

Fig. 1. Relative motion between chaser and target satellite on X-Y plane

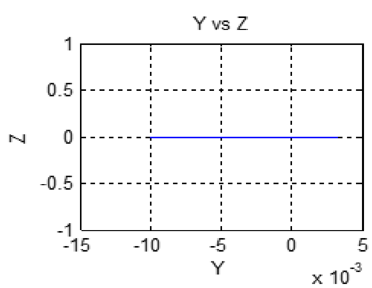

Fig. 2. Relative motion between chaser and target satellite on Y-Z plane

2 Clohessy Wiltshire Equations Incorporating J2 Disturbances [3]

Clohessy-Wiltshire equation do not incorporate J2 perturbations.CW equations incorporating the effects of J2 perturbations are discussed here.
The equation of motion of satellites around Earth is described by Equation (8).[2]

$$\ddot{\vec{r}} + \left(\frac{\mu}{r^3}\right)\vec{r} = \vec{f} \qquad (8)$$

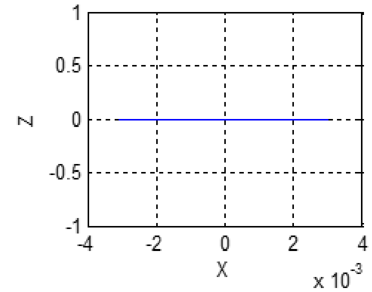

Fig. 3. Relative motion between chaser and target satellite on X-Z plane

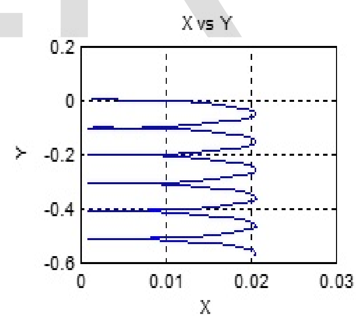

Fig. 4. Effect of perturbation on relative motion

Where μ is the Earth's gravitational parameter,$\vec{r}$ is the position Vector of satellite from Earth's center,$\vec{f}$ is the perturbing acceleration

The perturbing acceleration vector,[2]

$$\vec{f} = f_r\vec{e_r} + f_\theta\vec{e_\theta} + f_z\vec{e_z} \qquad (9)$$

Where$\vec{e_\theta}$,$\vec{e_r}$ and $\vec{e_z}$ are unit vectors along the radial vector direction, the transverse orbit direction, and the direction

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normal to the orbital plane respectively[2]

The six orbital elements are:

- The semi major axis(a),
- The eccentricity(e)
- The inclination(i)
- The right ascension of the ascending node(Ω)
- The argument of perigee(ω)
- The true anomaly (θ).

The changes in orbital elements of the LEO satellite due to J2 perturbation are represented by Equations(10)-(14) [2]

$$\Delta a = \frac{2a^2 p}{\sqrt{\mu p}}\{f_r e \sin\theta + f_\theta(1 + e\cos\theta)\} \quad (10)$$

$$\Delta e = \sqrt{p/\mu}\, P\{f_r \sin\theta + f_\theta(\cos\theta + \cos E)\} \quad (11)$$

$$\Delta\Omega = \frac{-3J_2R^2}{p}\cos i \int_0^{2\pi} \frac{(\sin(\omega+\theta))^2}{r} d(\omega+\theta) \quad (12)$$

$$\Delta\omega = \frac{-3J_2R^2}{2p^2}\left(\frac{5(\sin i)^2}{2} - 2\right) \quad (13)$$

$$\Delta\theta = \left(\frac{\sqrt{\mu p}}{r^2} - \frac{\Delta\omega}{p}\right)P \quad (14)$$

Where $P(\text{average orbital period}) = 2\pi\sqrt{a^3/\mu}$;

E–eccentric anomaly;

$p(\text{semi} - \text{latus rectum}) = a(1 - e^2)$;

$J_2 = 1082.64 \times 10^{-6}$;

R (radius of Earth) = 6378 km;

Then the components of perturbing acceleration are given by Equations (15)-(17)

$$f_r = \frac{-3\mu J_2 R^2}{2r^4}\{1 - 3(\sin i)^2(\sin(\omega + \theta))^2\} \quad (15)$$

$$f_\theta = \frac{-3\mu J_2 R^2}{2r^4}(\sin i)^2 \sin 2(\omega + \theta) \quad (16)$$

$$f_z = \frac{-3\mu J_2 R^2}{2r^4}\sin(2i)\sin(\omega + \theta) \quad (17)$$

Consider a circular reference trajectory of radius r_{ref} with origin at the center of Earth under the gravitational influence of spherical Earth.[3].Also consider two J2 perturbed satellites tracing circular orbits of similar altitude near to reference trajectory as shown in Figure.5 [3].Their equations of motion are given by Equations (18)-(20)

For the circular trajectory,[3]

$$\ddot{r}_{ref} = g(r_{ref}) \quad (18)$$

For the chaser satellite P1,[3]

$$\ddot{r}_1 = g(r_1) + J_2(r_1) \quad (19)$$

For the target satelliteP2,[3]

$$\ddot{r}_2 = g(r_2) + J_2(r_2) \quad (20)$$

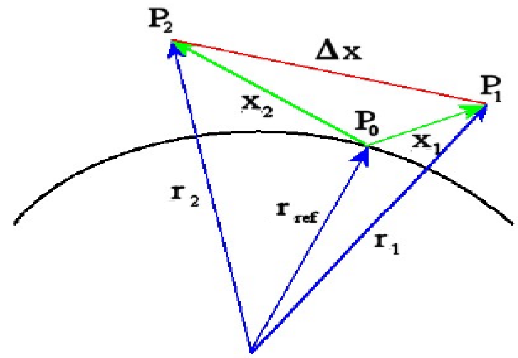

Fig.5. Relative motion of two perturbed satellites P1 and P2 [3]

Where,

$$g(r_{ref}) = \frac{-\mu}{r_{ref}^3}\overrightarrow{r_{ref}}$$

$$g(r_1) = \frac{-\mu}{r_1^3}\overrightarrow{r_1}$$

$$g(r_2) = \frac{-\mu}{r_2^3}\overrightarrow{r_2}$$

$$J_2(r_1) = \frac{3J_2\mu R_e^2}{2r_1^4}(1 - 3(\sin\theta)^2(\sin i)^2)\hat{x} + (2(\sin i)^2\sin\theta\cos\theta)\hat{y} + (2\sin i\cos i\sin\theta)\hat{z} \quad (21)$$

$$J_2(r_2) = \frac{3J_2\mu R_e^2}{2r_2^4}(1 - 3(\sin\theta)^2(\sin i)^2)\hat{x} + (2(\sin i)^2\sin\theta\cos\theta)\hat{y} + (2\sin i\cos i\sin\theta)\hat{z} \quad (22)$$

The relative accelerations of the perturbed satellites P_1 and P_2 relative to the reference trajectory, considering the reference orbit frame rotation with angular velocity ω on the reference orbit, are

$$\ddot{x}_1 = \ddot{r}_1 - \ddot{r}_{ref} - 2\omega \times \dot{x}_1 - \dot{\omega} \times x_1 - \omega \times (\omega \times x_1) \quad (23)$$

$$\ddot{x}_2 = \ddot{r}_2 - \ddot{r}_{ref} - 2\omega \times \dot{x}_2 - \dot{\omega} \times x_2 - \omega \times (\omega \times x_2) \quad (24)$$

Where ω is the velocity vector of a satellite tracing the reference orbit .After some substitutions and by subtracting the relative motion equations (23) and (24),the Equations (25)-(27) are obtained:[3].The Equations(25)-(27) represents equations of motion in the radial, in-track, out-track directions.

$$\Delta\ddot{x} - 2nc\Delta\dot{y} - n^2\left[c^2 + 2 + \left(\frac{6J_2R_e^2}{r_{ref}^2}\right)(1 - 3(\sin i)^2(\sin\theta)^2\right]\Delta x - \left(\frac{6J_2n^2R_e^2}{r_{ref}^2}\right)(\sin i)^2\sin(2\theta)\Delta y - \left(\frac{6J_2n^2R_e^2}{r_{ref}^2}\right)\sin(2i)\sin(\theta)\Delta z = 0$$

$$\Delta\ddot{y} + 2nc\Delta\dot{x} - \left(\frac{6J_2n^2R_e^2}{r_{ref}^2}\right)sin^2 i\sin(2\theta)\Delta x - n^2\left\{c^2 - 1 - \left(\frac{6J_2R_e^2}{r_{ref}^2}\right)\left[\frac{1}{4} + sin^2 i\left(\frac{1}{2} - \frac{7}{4}sin^2\theta\right)\right]\right\}\Delta y + \left(\frac{6J_2n^2R_e^2}{r_{ref}^2}\right)\left(\frac{\sin(2i)\sin(\theta)}{4}\right)\Delta z = 0 \quad (26)$$

$$\Delta\ddot{z} - \left(\frac{6J_2n^2R_e^2}{r_{ref}^2}\right)\sin(2i)\sin(\theta)\Delta x + \left(\frac{6J_2n^2R_e^2}{r_{ref}^2}\right)\left(\frac{\sin(2i)\cos(\theta)}{4}\right)\Delta y + n^2\left[1 + \left(\frac{6J_2R_e^2}{r_{ref}^2}\right)\left(\frac{3}{4} - (\sin i)^2\left(\frac{1}{2} + \frac{5}{4}(\sin\theta)^2\right)\right)\right]\Delta z = 0 \quad (27)$$

J_2- Second harmonic due Earth oblateness effect;

$n = \sqrt{\frac{\mu}{r_{ref}^3}}$ (Angular velocity of satellite tracing reference orbit);

R_e – radius of the Earth;

r_{ref} –Radius of reference orbit;

i –Inclination;

θ –True anomaly;

$\omega = n\hat{z}$ (Velocity vector of satellite tracing reference orbit);

$c = \sqrt{(1+s)}$

$s = \frac{3J_2R_e^2}{8r_{ref}^2}(1 + 3\cos 2i)$.

On further simplification of Equations (25)-(27), the Equations (28)-(30) are obtained

$$\ddot{x} - 2nc\dot{y} - (5c^2 - 2)n^2x = 0 \quad (28)$$

$$\ddot{y} + 2ncx = 0 \quad (29)$$

$$\ddot{z} + (3c^2 - 2)n^2z = 0 \quad (30)$$

Where $x, y, z, \dot{x}, \dot{y}, \dot{z}$ and $\ddot{x}\ \ddot{y}\ \ddot{z}$ are the relative distances, relative velocities and relative accelerations between the two satellites in radial, along-track and cross-track directions.

The equations in state-space form are given in (31) [3]

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ (5c-2)n^2 & 0 & 0 & 0 & 2nc & 0\\ 0 & 0 & 0 & -2nc & 0 & 0\\ 0 & 0 & -(3c-2)n^2 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ \dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} \quad (31)$$

The state space equation, modified with control inputs is given in (32).

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ (5c-2)n^2 & 0 & 0 & 0 & 2nc & 0\\ 0 & 0 & 0 & -2nc & 0 & 0\\ 0 & 0 & -(3c-2)n^2 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ \dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} + \begin{bmatrix}000\\ 000\\ 000\\ 100\\ 010\\ 001\end{bmatrix}\begin{bmatrix}u_x\\ u_y\\ u_z\end{bmatrix} \quad (32)$$

The J2 perturbations will affect the relative distance between the chaser satellite and target satellite. Therefore three control inputs are added which will maintain the radial, in track, out-track relative distance component[x y z] constant. Take radius of the reference circular orbit r_{ref}=7000km (LEO), inclination=35.

Earth Gravitational constant 6.673×10^{-11}N·m²/kg², Mass of the Earth (M) =5.98×10^{24}kg, J2= 1082.64×10^{-6}.By using these values, the states space equation becomes (32) as in Equation (33)

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} \begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0.0022 & 0\\ 0 & 0 & 0 & -0.0022 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} = \begin{bmatrix}x\\ y\\ z\\ \dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}u_x\\ u_y\\ u_z\end{bmatrix} \quad (33)$$

2 LINEAR QUADRATIC REGULATOR

Linear Quadratic Regulator is a state space based optimal control method. Consider a linear, continuous-time and controllable system and the performance criteria as described in equation (34) and (35) respectively

$$\dot{x} = Ax + Bu \quad (34)$$

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$$J = \int_0^\infty [x^T Qx + u^T Ru]dt \qquad (35)$$

Where Q and R are weighting matrices and should be positive-semi-definite and positive definite, respectively is the cost function.LQR control problem is to calculate function $u(t) = -Kx(t)$such that cost function is minimized and R are controller design parameters that penalizes the states x and the control effort u respectively.

The LQR controller has the following form

$$u(t) = -Kx(t) \qquad (36)$$

$$K = -R^{-1}B^T P \qquad (37)$$

Where P is given by the positive (symmetric) semi definite solution of

$$0 = PA + A^T P - Q + PBR^{-1}B^T P \qquad (38)$$

The equation (38) is called Ricatti equation. It is solvable iff the pair (A,B) is controllable and (Q,A) is detectable. The solution of Ricatti equation and computation of K can be done in MATLAB.An initial Q and R matrix are chosen. Then R is kept fixed and Q is varied to get good response. By changing the value of Q and running the command [K]=lqr(A,B,Q,R),

K=14.1421	-0.0049	0.0000	6.1874	0.0000	0.0000
0.0049	14.1421	0.0000	0.0000	6.1874	0.0000
0.0000	-0.0000	12.2474	0.0000	0.0000	5.8732

The simulation of step response of system using this controller is shown in next section

2 Results And Discussions

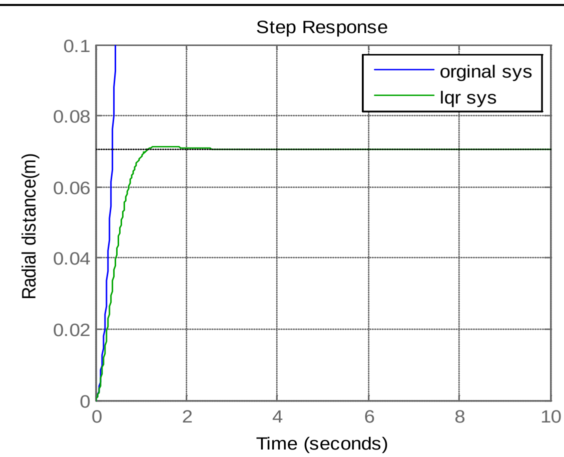

Fig. 6.Step response with LQR controller in radial direction

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lqr

sys..Figure(6)-(8) also plots the open loop response depicted as **original sys**.Figure shows that LQR controller provides effective control to radial,intrack,out-track distance ,thus helps to maintain constant relative distance between the chaser and target satellite. The performance of LQR controller is shown in the Table 1.

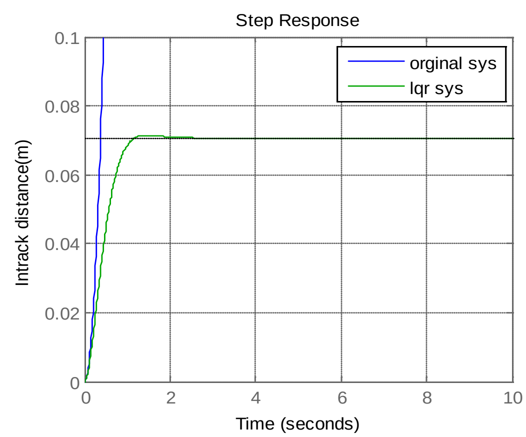

Fig. 7.Step response with LQR controller in in- track direction

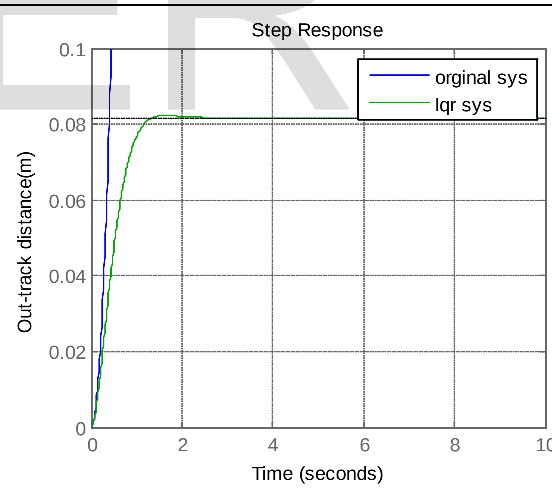

Fig. 8.Step response with LQR controller in out-track direction

3.1 Adding a precompensator

From this Figures (6)-(8), note that the steady state error is too large.one method for eliminating it is to scale the reference Input so that output in turn scaled to desired level. Step response plots with LQR controller and precompensator is shown in Figures (9)-(11).Note that the steady state error is reduced by adding the Precompensator.

TABLE 1
PERFORMANCE SPECIFICATIONS

Performance parameters	Radial	In track	Out-track
rise time	0.6796 s	0.6796 s	0.7492 s
Settling time	1.0485 s	1.0485 s	1.1683 s
overshoot	1.0609	1.0609	0.7856

s = second

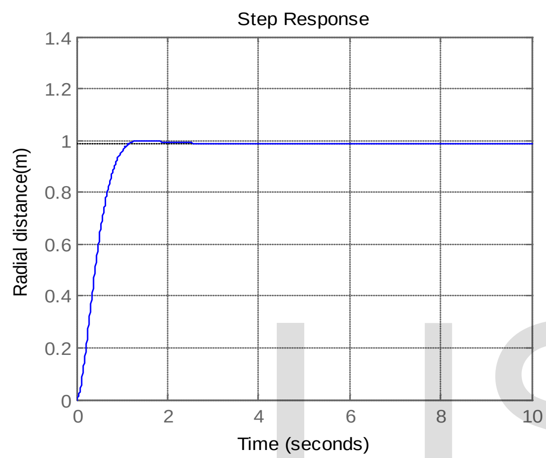

Fig. 9.Step response with LQR controller and precompensator in radial direction

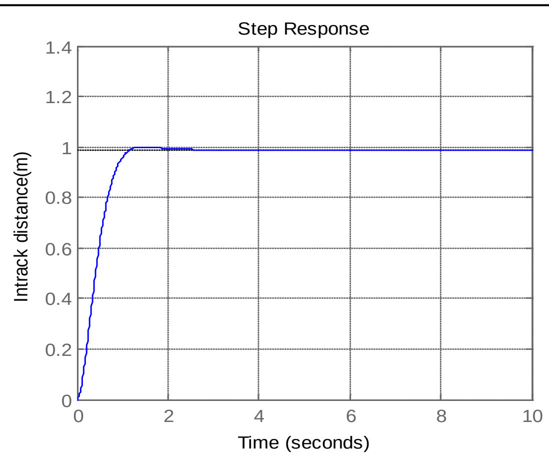

Fig. 10.Step response with LQR controller and precompensator in in track direction

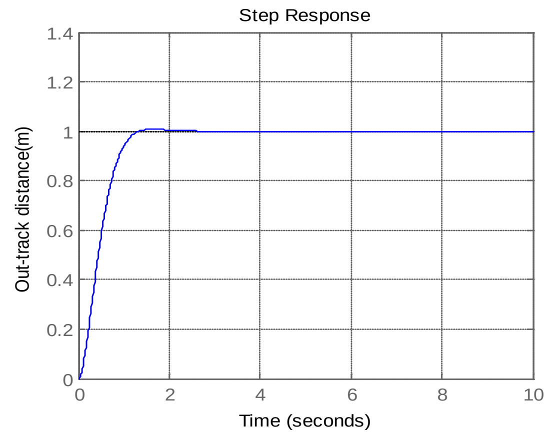

Fig. 11..Step response with LQR controller and precompensator in out-track direction

5 CONCLUSION

Relative motion among the two satellites is plotted in MATLAB using C-W equations. Modelling of relative motion among satellites under J2 perturbations were discussed. An LQR controller is designed and simulations are done in MATLAB.It is found that LQR provides better control to maintain the relative distance constant among the two perturbed satellites The future work can be implementation of LQG control and compare the results.

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